

ASSOCIATING A MATHEMATICS MODEL TO A REAL SITUATION WITH GEOGEBRA

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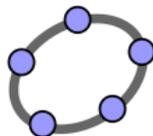
ABSTRACT

This activity was thought and elaborated with the goal of showing a proposal of work, with the utilization of a software, to a group of teachers, during a postgraduate course, “Math and New Technologies”, they are in a process of changing their posture up their students, making them to reflect about their teaching practice in the moment that the focus in the education is not to develop reading skills, writing or basic calculation anymore. The used resources were: GeoGebra software, block of activities, a computer and a multimedia projector. They were applied in two moments to different groups with the purpose of better evaluate the obtained results. Initially in a public school in Salvador – Bahia with 5 students from the 2nd grade of the high school and subsequently with 2 math teachers, from public schools. The applied methodology was based in the resolution of the problem where the investigative posture by all involved will always exist. The experiences mentioned were enriching to analyze concepts and definitions based in a mathematic model reconstructed from an object that makes part of the student’s reality.

Keywords: Technology. Learning. Investigation. GeoGebra.

INTRODUCTION

In the first years of the 20th century, the education had as its focus the acquisition of learning abilities, writing and basic calculation. The rule was to practice and not to critically think or read. The abilities no longer serve the needs of the current society. Before, the school was our biggest source of information and the model was to give instruction, order, control, supervise, evaluate, create stars; today is to inspire, involve, capacitate, give support, create teams, learn and innovate.



It is necessary to rethink the educational training where the teacher might be a mediator of the dialogue of the teaching with the knowledge. The educator must have the necessary orientation to the knowledge object be explored by the students, without offering them the ready solution. Therefore, it is necessary to modify it having as a reference and actions that deal with the learning in a significant way to the students in a social and educative perspective.

The teacher's function is to stimulate the students; so they relate the ideas, have autonomy of thinking, discover, create, think and ratiocinate. For that, the teacher must work with ideas, intuitive concepts, making the student to learn by comprehension. Work by means of problem situations, which come from the student's experience and make them really think, analyze, judge, decide for the best solution; causing that the worked content be important.

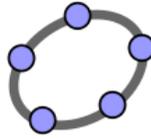
According to POLYA (2006), the teacher must challenge the student's curiosity with problems that are in agreement with his/her level of knowledge helping them with questions that stimulate them to think, ratiocinate, motivating them.

In agreement to the National Curricular Parameters (BRASIL, 2000), one of the ways of teaching math is to provide the teachers opportunities to have the experience of situations similar to the reality which is around us.

In this context the idea of searching initially a visual stimulus appeared, to cause impact, a box of French fries from McDonald's, with the purpose of involving everybody and create a favorable scenario to a work proposal where the investigation could be around during the entire process.

The problem appeared after the invitation: Let's draw a box of French fries of McDonald's which can be found in the block of activities that was given, using GeoGebra?

This activity was performed in two moments. Initially it was applied to a group of students, the total was 5, from the 2nd year of the high school, they were chosen among the best students in math and they have had experiences with the GeoGebra software and the applied methodology before; the used time was 4 classes. In the second moment the activity was performed by two teachers who did not have a lot of experience with the applied methodology but they knew GeoGebra. The idea was to present a new work proposal which could attend the demands of the current society. They constructed separately the box of French fries, analyzing the possibilities and possible solutions.



In the first moment, the activity was performed with a group of students, in 4 steps.

Step 1 – It was performed in 60 minutes, where the students received the material and the information about the necessary procedure. After the instructions; they requested a quickly review about the subjects that would be worked.

Step 2 – They started the activity, they requested permission to consult books and notebooks besides of socializing each one of the discoveries and the constructed steps.

Steps 3 and 4 – They concluded the box and they investigated a way of drawing the supposed French fries.

The second moment – The teachers started the construction of the model, writing down the possible questionings and doubts that could appear and they finished in their houses sending the contribution subsequently.

DEVELOPMENT

Step 1 – Initially it was distributed a notebook of activities where the students should write down the discoveries, doubts and conclusions made from the study that would be performed. There was not any rigidity in a way to force the students to answer some questions approached in a determined sequence. It was elaborated by the teacher and it appeared in the cover of a box of French fries from McDonald's. They all got surprised and lost because they did not know what to do and neither why there was that picture on the cover. They had constructed the McDonald's logotype with the teacher in another opportunity but they could not think about any possible relations that they could do with the previously reviewed content and that drawn box.

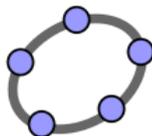
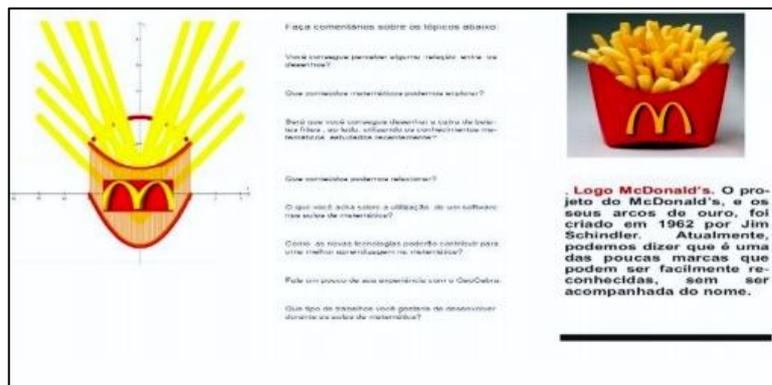
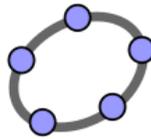


Figure 1 – Block of activities



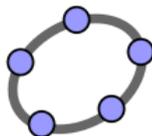
Source: Prepared by the authors

The students received the notebook with the activities, they showed surprise but they did not have the initiative in the sense of starting the proposed activity. In front of this students' posture the teacher decided to start a dialogue with the purpose of helping them to think. (T) – Observe the drawing that can be found in the activity block, what do you see? (S) – The box of French fries from McDonald's! (T) – Correct! What mathematical contents could we explore? As the teacher did not obtain any answer she decided to continue with the questions. (T) – Imagine this box being drawn in a Cartesian plan! What studied and reviewed contents could we associate? (T) – Think in the recently reviewed contents! What relation will we have been done? (S) – We studied functions, graphs, locations of points, (T) – Great! What's the relation could we do with each of these topics and the drawing on the box? Think! When you refer to the points, can we approach the localization of the points on the plan? What that would help me so I can draw? (S) - When I want to put it in some place. (T) – Do you refer to a determined place? A specific place? (S) – Yes, when I want to draw on some place I say where. (T) – Great! Correct! (T) – So let's make a choice? Where would you like to draw? Give me a suggestion! (S) – The right side. (S) – On the top. (S) – More to here. (T) – Well, imagine you are doing this work with someone who on the other side of the line, in another place! Would this person understand this way? (S) – Teacher, on the 1st quadrant! (T) – Great, excellent, do you all agree? (S) – Can be. (A) – I prefer in the middle, look at the box teacher! (T) – This is going to change, in a sense of facilitate? (S) – I don't know; it must be a reason. I think that, if she drew in the middle it's because it must be easier. (T) – Imagine when we were analyzing the graphs, the displacement of the graph was related to what? (S) –



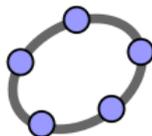
Vertices, roots and the value of c . (T) – Well, are you thinking in a parabola? (S) – Yes. (T) – So let's continue with this idea. (T) – When we make a choice about the localization of the vertex and the roots so we can draw a parabola, in two different places, for example, 1st and 2nd quadrants. What does it change? (S) – The place of the parabola. (S) – Teacher, can you review this graph topic? Before we continue the drawing. (T) – Of course! Let's do a quickly review about the functions and then we will return to our activity.

Step 2 – In this moment; it was clear that the students returned to the activities with more determination and involvement. They seem to be ready to face the challenge and construct the model, and for that, they took a book to consult. They asked for permission to consult and I allowed to all of them; the right of researching in books and notebooks. Continuing the activity of the last class, it was started a necessary dialogue to better analyze and construct the model exploring the possible contents to be worked at the moment. (T) – Let's decide the local to trace the parabola? (S) – I want equal to the example. (T) – Right, but let's suppose that I want you to determine the algebraic representation of the function that gave origin to the graph you want to plot. Will the level of difficulty vary? (S) – I think so. (T) – You think? (S) – Ah! I remember that when c was equal to zero, it was easier. (T) – Easy how? (S) – When you asked something that was need from the formula, the calculation, it was easier when c was equal to zero. (S) – In the 1st degree it was also easier when b was zero. (T) – Well, let's think in these two examples. In the 1st example you refer to a polynomial function of the 2nd degree where $f(x) = ax^2 + bx + c$, right? You affirmed that when $c=0$, the calculation to find the algebraic representation of the function was easier. Let's verify the meaning of this? What does it happen with the graph of a function when the coefficient is $c=0$? (S) – I don't remember this way. (S) – Can we research? (T) – Sure. (S) – Can we look for examples in the book and in the notebook? (S) – I found an example! (T) – Well, now think what happens with the graph! (S) – There is here in the notebook that it is the point where the graph cuts the y axis, I'm remembering teacher! (T) – Great! You're saying that, the coefficient c of the algebraic representation of the function; indicates the value of the coordinate y , in the point where the parabola cuts the axis of the ordinates. Observe these examples! Think, what are the values of c ? (S) - $c=2$ and $c=0$. (T) – Great! What does it change? (S) – The place which is in the parabola changes. (T) – Wonderful! That's right! (S) – Is it going up or down? (T) – Let's think in the second example where we had a function.

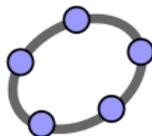


Think in what your classmate said, he remembered the coefficient b . Imagine what happens with a graph of a function when coefficient b varies! Imagine $b=2$ and then $b=0$. (S) – Teacher, b is where the straight line cuts the y axis too. (S) – Teacher, in the parabola is the c and in the straight line is the b . (T) – Let's observe what our classmate concluded. In the polynomial function of the 2nd degree $f(x)=ax^2+bx+c$ and in the polynomial function of the 1st degree, $g(x)=ax+b$, the independent terms are b and c . When we analyze the graphs we realize that the ordinates of the points are where the graphs cut the y axis (ordinates). They are important information. (T) – How come can these information interfere in our study? In the drawing? (S) – There is a straight line going by the zero and b is zero. (T) – Where are you visualizing the straight lines? (S) – On the French fries. (T) – Ah! You are planning to draw the French fries using the equation of the straight line, right? (S) – That's right, and if I need a and b , when I put $b=0$, it gets easier. (T) – Excellent! Observe that the position might facilitate or complicate our work. How about we divide into two groups, where a group would draw how it is in the model and the other would choose any another position? (S) – If everybody helps, I'm in. (T) – Great! We will start in the next class.

Step 3 – After the accomplished questions during the last class; the students took the activity home with the goal to think in possibilities on construction. All of them had the program installed in their computers and the proposal was to try to construct individually before our next class. Initially we had a moment where all of them had the opportunity to exhibit their difficulties and doubts so the teacher could clarify and orientate them. After this moment they started the construction trying to draw exactly how it was in the activities block. (T) – Well, thinking in the polynomial function of the 2nd degree, what do you observe? (S) – If the straight line didn't end here, it could be a parabola. (T) – Where did you identify a possible parabola? (S) – Here, in this part of the box. (T) – Let's try? If we wanted to plot a parabola to represent this part of the box, what should we do? (S) – We should know where the vertices and the roots are. (T) – Great! Could you find these points for me? (S) – The vertex is easy, but how am I going to know the roots if the straight line doesn't cut the x axis? (T) – Imagine a parabola going by the vertex that you suggested and cutting the abscissa axis. (S) – But the part of the box that cuts this axis is not from the parabola. (T) – Imagine that I only used this part of the box. (S) – What did you do with the rest of the parabola? How can we do this? (T) – Well, I imagined a function with a hole similar to this part of the box,



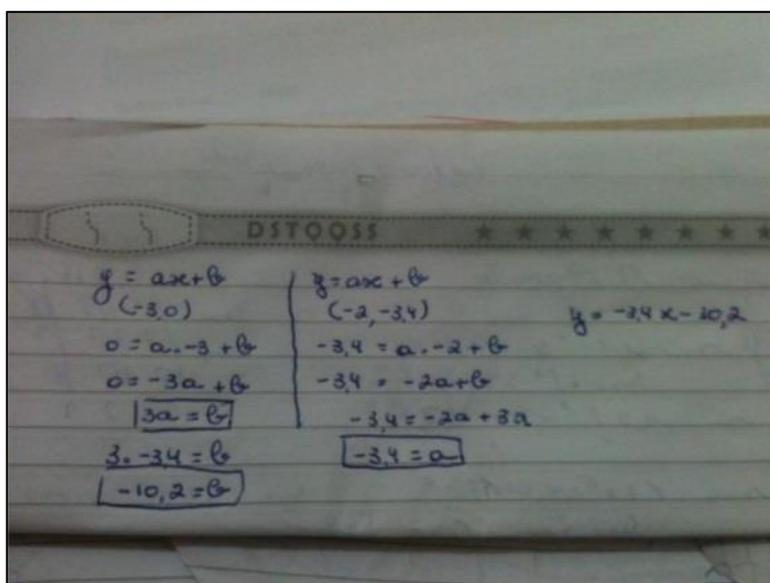
concavity up, the axis of symmetry coinciding with the axis of the ordinates and the coefficient c is equal to -4 . (S) – The big hole where a gets really small, right teacher? (T) – Can you exemplify? (S) – Can be $F(x) = x^2 - 4$? (T) – Let's try? (S) – It's not good, we need to open more. A lot of examples appeared and we chose $f(x) = 0,2x^2 - 4$. During this moment a lot of questions appeared around the value that would be attributed to the coefficient b , when we say that $b=0$ the axis of symmetry of the parabola coincide to the axis of the ordinate. It was a very important moment where everybody participated; they gave examples and took their own conclusions in a satisfactory way after viewing lots of examples, analyzing each plotted graph. They typed the chosen function and after they observed the graph the problem was around what they should do; so the parabola could be limited. In this moment the teacher invited the group to think in how the graph is plotted. (T) – If we were going to plot the graph on a paper with a pencil, what should you do at first? (S) – The table. (T) – Correct, to the table be elaborated we should attribute numbers to the variable x , do you agree? (S) - Yes and then find y . (T) – Great! That's it! Observe that the number we attributed to the variable x is the one that determines. (S) – The domain is with x . (T) – Excellent! We don't need to limit this group. We limited the domain. (S) – Is the straight line going to stop where we want? (T) – Yes, the straight line must be limited; it goes where you want to. (S) – Now it's easy, teacher! (T) – Great! So, let's continue? (S) – How are we going to do at GeoGebra? In this moment the teacher stopped to give the necessary information. The domain was limited as $D(f) = [-2,2]$, a new function appeared: $g(x) = 0,2x^2 - 4$ and the students were orientated to click in the function: $f(x) = 0,2x^2 - 4$ so the same one could be hidden and they would only paint what was necessary to form the box. The next steps were to find a function where the graph could cut the axis of ordinate in one point between -4 e -3 , a hole a bit bigger than the last one plotted and concavity down. Initially the chosen function was $h(x) = -0,2x^2 - 3$ and subsequently they realized that they should enlarge the hole reducing the value of a . Consequently they had to do some alteration in the function $f(x)$ and this one became $f(x) = 0,1x^2 - 3,7$, limited $h(x)$ and this one became with the domain equal to $[-2, 2]$; and after that it hid $h(x)$. The next step was to think how it could be represented the side of the box and all of them thought automatically in straight lines. We started the study of the function thinking in plot two straight lines that could attend the need of the moment. It was requested to the students that they should review at home thinking to conclude the activity next class, as



planned initially.

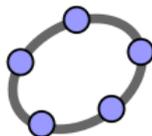
Step 4 – The 4th and last step was started with the students choosing two points so we could plot a straight line with the goal of drawing the side of the box. They analyzed which points would be the best choice and they chose these ones: (-3,0) and (-2, 3,4). They made the due replacements as the picture below:

Figure 2 – The need calculation to find the function



Source: Prepared by the authors

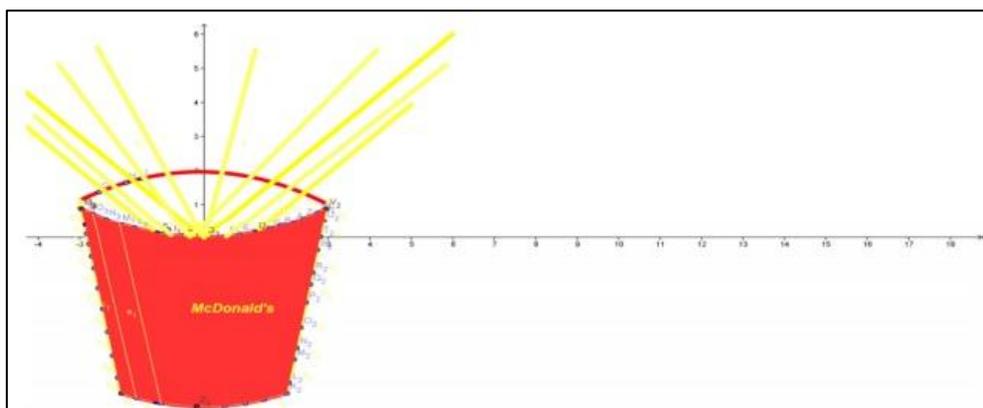
The chosen function was $y = -3,4x - 10,2$. After they plotted the straight line they realized the need to limit the domain and they chose the Domain = $[-4, -2]$. Afterwards the teacher requested them to hide the function with the domain \mathbb{R} , previously plotted. The next step was to plot another straight line and do the exactly procedure. The chosen points were: (3, 0) and (2, -3, 4). They made the due replacements and they drew a conclusion that the function that should be plotted was $y = 3,4x - 10,2$. The domain was limited, it was equal to $[2,4]$ and previous function that was typed, it was hidden. The next step was to plot a parabola that attended the needs to draw the other part of the box. The students analyzed the presented situation, so they could think in a function that attended to this new situation. They concluded that the parabola should cut the axis of the ordinates in 1 and for that they should make that the value of c , was attributed this value, and the value of a should be 0,2 to coincide with the



opening of the one parabola plotted previously. They chose to limit the domain, now it is: $D(f) = [-3,5, 3,5]$. After all these procedures they chose to name four points that would be connected by segments established by two points, so now the box could be closed. Finally, they tried to paint the box and they were informed that GeoGebra only paints polygons. They contoured the box so it could be painted and then they drew the French fries. An idea appeared; they wanted to form the French fries with segments or straight lines, but the proposal was to work with functions; thinking in a lot of possibilities; the teacher suggested they should plot in base in the knowledge in modular function. The goal of conducting the activity choosing to work with modular function was to the students to analyze the graphs observing the dislocation in function of the choices and the wanted positioning. During the lived experiences because of the made choices they reach a few conclusions: when $y = |x| + 1$ the graph climbs 1, even $y = |x| - 1$ the graph climb down 1, to $y = |x+1|$ the graph moves 1 to the left, to $y = |x-1|$ the graph moves 1 to the right, to $y = |x+2|$ the graph moves 2 to the left, this was is clear to all the students.

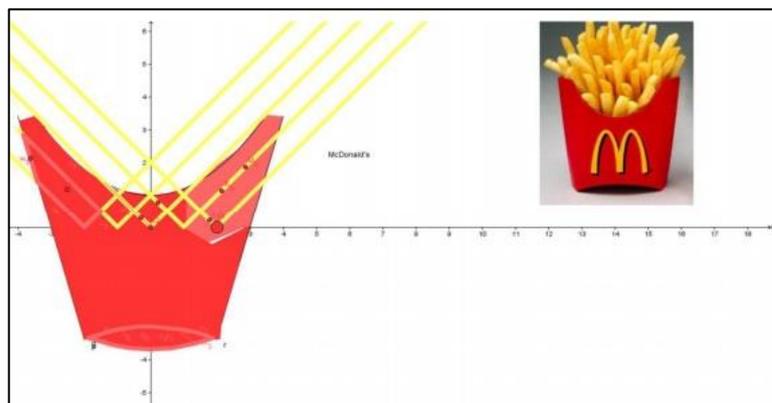
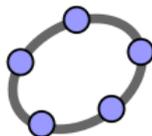
After they drew the box and the French fries, they decided to put the name using one of the commands of the program and they saved in the Word as the pictures below.

Picture 3 – 1st box constructed



Source: Prepared by the authors

Picture 4 – 2nd box constructed with the teacher's presence.



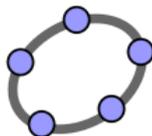
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In the second moment of the activity it was performed by two teachers who had the opportunity to have the experience moments where the computer does not replace the human being to commit less mistakes, it does not complement the human being in the sense of execute some parts and the man others, on the basis of the supplementation theory, but it provokes, reorganizes, models and it is modeled. The proposal puts the teacher as a protagonist of his/her own process of learning and learn by comprehension. To these teachers it was given the activities block but it was solicited that both of them think about a way of evaluating the students with basis on the presented proposal, where they need to conduct this construction with their students. They presented the proposal below and they drew the box with the orientation of the teacher who idealized the work.

Make this parabola to have its concavity up and the zeros of the function which gave the origin must be +2 and -2. Attribute to the coefficient a value that attends the conditions above. Make in a way that the graph intercepts the axis of the ordinates on the point (0, -2).

After this step, answer the following questions:

- What's the axis of symmetry, the coordinates, points minimum and maximum, domain and image and codomain. Justify your answer making correlations with the formulas which are used and the graphic representation of the function.
- To the parabola you have just plotted, make that the domain be $[-2,2]$. What did you understand? Analyze the change that happened.



- c) Plot a new parabola where $xv=0$, the hole two times bigger than the last one and the coefficient $c=1,0$. In these conditions, what's the algebraic representation of this new parabola? What do you conclude from the positioning of a parabola that has $xv=0$?
- d) What does it mean to say that the hole of a parabola can be twice another one? When this happens, what can it interfere in relation to its coefficients?

FINAL CONSIDERATIONS

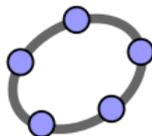
The studied theories evolving the new technologies question their use as an instrument, tool or element which causes a new thinking. According to Tikhomirov; there are three theories about how the computers affect the human cognition: The first theory is the Replacement where the computer is seen as a substitute of the human being, the thinking is trivial, ignoring the complex humans' process, the second is the Supplementation, based on the theory of the information where the computer complements the human being and the thinking can be divided into small parts where the men perform some of them and the computer performs others, resulting all that before was performed just by the human and the third theory is Reorganization where it provokes a reorganization in the human activity.

After the introduction of the new technologies, some concerns occurred referring to the curricular changes, new dynamics in the classroom, the teacher and student role.

LÉVY (1993) highlights the importance of the media to the human thinking, when he says: “ the libraries and the new interfaces of the computers are not just frames” , but yes an active part of the thinking. Our thinking, although it is not determined but it is conditioned by the different techniques developed over the History, because the use of the media provides that experiments can be done, enlarging the possibilities.

CONCLUSIONS

The use of the computer with GeoGebra involved the students; giving the opportunity to try to discuss discoveries and arouse curiosities. Some conjectures could be associated to the large experimentation done which is only possible due to its use, enlarging possibilities, providing experiments. We can determine the importance of the Medias' role and specifically



of the informatics in the math thinking of students who use this information technology next to other resources. According to TIKHOMIROV (1981, apud BORBA; PENTEADO, 2001), the human thinking is reorganized when the informatics is incorporated to the students' routine.

In Mathematics, the way it is taught and learned has been very discussed, driving to the reflection about new teaching proposals by renovations in the educational practice; however, it's realized that the math teaching is based almost uniquely in the transmission of knowledge and not the construction (exposition and accumulation of formulas, algorithm and application of rules).

The students feel a big difficulty in learning mathematical concepts because they do not realize what's the use of them and increasingly they are away of the reality. It is possible to improve the situation; giving the students the opportunity to develop the process of construction of knowledge, by impulse with activities and potentially powerful environments, that might provide the mathematic learning.

The new challenge is to implement the approximation between the teacher and the student, putting them as co-authors of the performed activities by the collaborative learning. The computer is an important tool that can be a powerful technology in resources to be applied in the education with the goal to guarantee the interaction man-machine as an essential element to learning.

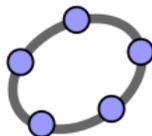
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